

# 록인 앰프 기반의 주파수 잠금 루프와 적응형 분해능을 갖는 최적 윈도우 FFT를 이용한 새로운 고조파 검출 알고리즘

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## A Novel Harmonic Detection Algorithm using Optimal Windowed FFT with an Adaptive Resolution and Lock-in Amplifier Based Frequency-Locked Loop

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### Abstract

The world has experienced a surge in the usage of grid-connected systems such as renewable energy systems and electric vehicle charging stations. These grid-connected systems and nonlinear loads, disrupt the normal grid network operation which results in reduced power quality as a result of non-sinusoidal grid conditions like harmonics and frequency deviations. The reduced quality of the grid power results in inaccurate harmonic analysis and power measurement which leads to inefficient management and planning in the power sector. To solve these problems this article proposed a novel harmonic detection method using the optimal windowed FFT (OWFFT) with an adaptive resolution and the Lock-in Amplifier Frequency-Locked Loop (LIA-FLL). The validity of the proposed method is verified by experimental implementation using NI myDAQ and LabVIEW, comparing the results with currently used methods (Hanning Window Interpolated FFT, Hanning window double-spectrum-line Interpolation FFT, and Nuttall double-window FFT).

### 1. Introduction

The recent expansion in the network of the distributed power generation (DPG) system and electric vehicle charging stations results in increased grid capacity. In addition, international treaties like Paris Climate Agreement are forcing the government to promote electric vehicles (EVs) and DPG systems. This increase in grid capacity and the trend of EVs, DPGs, and non-linear loads are affecting the power quality in the grid by the injection of harmonics and causing frequency fluctuations. The decreased power quality results in decreased safety, overheating, system losses, and reduced stability of the system. This non-reliability in system operation results in inefficient operation and poor planning for the mitigation of hazards [1].

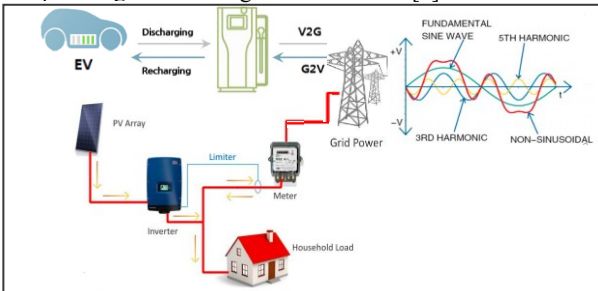


Fig. 1. Grid-Connected Distributed Power Generation System and Electric Vehicle Charging Station.

Fast Fourier transform (FFT) is the most widely used method for harmonic analysis in the power system. FFT is an improved version of Discrete Fourier transform (DFT) with less number of

operations to get the results. It eliminates unnecessary calculations using the properties of symmetry and periodicity.[2] The FFT method results in accurate harmonic analysis in the case of synchronous sampling. However, it has some limitations such as spectral leakage and picket-fencing effect. These effects occur when there is an oscillation in the frequency of the input signal and it is not possible to sample the signal synchronously and the truncation of the data set length has a non-integer period. This results in inaccurate parameters like amplitude, frequency, and phase. Figure 2. Shows the comparison between the FFT results with 1Hz resolution and input signal having an amplitude of 1V. It can be seen that first, the input signal frequency is 20Hz the amplitude extraction is accurate. However, as the input signal frequency changed to 20.4Hz (Non-integer multiple of resolution) the amplitude extraction result gets inaccurate [2,3].

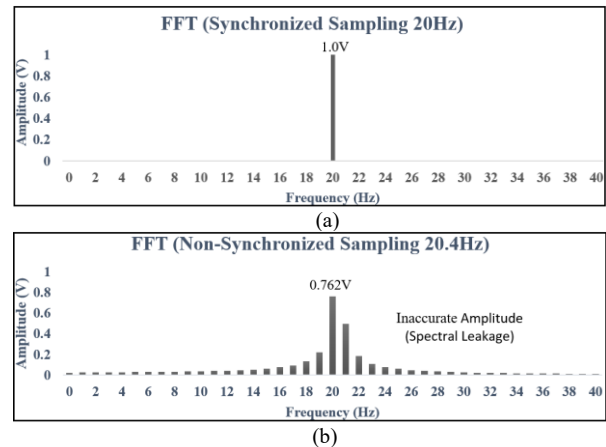


Fig. 2. FFT results with (a)No Spectral Leakage, (b)Spectral Leakage

To resolve the limitations of FFT, two main solutions are currently being used. One solution is the introduction of windowed FFT (WFFT), in which the window function reduced the side lobe amplitude and converted the sampled signal to be periodic. There are different types of windows proposed like Hanning, Hamming, Black, Blackman-Harris, Blackman-Nuttall, Flat-Top, Triangular, and Welch. There is no specific window so the window is selected based on the application. The selection is based on the width of the main lobe, amplitude, and roll of rate for side lobes. The second solution being used is the interpolated FFT (IFFT), in this method, the spectral lines (Double-spectrum-line and Triple-spectrum-line) with maximum amplitudes are used to calculate the amplitude, phase, and frequency correction factors. A combination of these methods is also proposed such as windowed interpolated FFT (WIFFT). The application of such methods resulted in the improvement of the FFT implementation. However, these methods still show some errors, especially with high-frequency bandwidth, and with the increase in the

technology and requirement for accurate results, new methods with higher accuracy are required for further improvement in the harmonic analysis of the power system [1-3].

This article proposes a novel power harmonic detection algorithm using optimal windowed FFT (OWFFT) with an adaptive resolution and Lock-in Amplifier Frequency-Locked Loop (LIA-FLL). The LIA-FLL tracks the input signal frequency deviation. The OWFFT uses this tracked frequency to find the optimal resolution and gives a spectrum with amplitude and phase information. The validity of the proposed method is verified by comparing the results for the proposed method with Hanning window-based WFFT, Hanning window double-spectrum-line Interpolated FFT, and Nuttall double-window FFT.

## 2. Proposed Method

The proposed method is based on the OWFFT and LIA-FLL. The LIA-FLL tracks any change in the frequency of the input signal and the OWFFT with optimal resolution extracts the amplitude and phase information.

The LIA blocks show that there are three sections. In Phase Sensitive Detection (PSD) the input signal is multiplied with two separate reference signals to implement dual modulation Lock-in strategy. This operation locks the specific frequency component. The output of the PSD consists of the DC and double frequency component of the reference signal. The DC component has the phase and amplitude information of the targeted frequency component. Equations (1), (2), and (3) show the voltage signal with multiple frequency components and two reference signals.

$$V_{sig} = V_{amp} \begin{pmatrix} \sin(\omega_f t + \theta_f) + \\ m_2 \sin(2\omega_f t + 2\theta_f) + \\ \dots \\ m_n \sin(n\omega_f t + n\theta_f) \end{pmatrix} \quad (1)$$

$$V_{sin\_ref} = \sin(k\omega_{ref}t + k\theta_{ref}), \quad (2)$$

$$V_{cos\_ref} = \cos(k\omega_{ref}t + k\theta_{ref}), \quad (3)$$

Where “ $V_{amp}$ ” is the peak amplitude of the fundamental component, “ $m_n$ ” is the amplitude ratio of the  $n$ th harmonic, and “ $\omega_f$ ” and “ $\theta_f$ ” is the frequency and phase information of the fundamental component respectively. The ‘ $k$ ’ defines the order of the reference signal selected according to the harmonic component to be extracted. The ‘ $\omega_{ref}$ ’ and ‘ $\theta_{ref}$ ’ is the frequency and phase of the reference signals, respectively [4].

To extract the DC component and eliminate the double frequency component the outputs of the PSD section are filtered out using Low Pass Filters (LPFs). The output of the LPFs is used to calculate the phase information for the selected frequency component. The LIA-FLL uses the phase output ( $\theta_n$ ) of the LIA to find the error frequency ( $\Delta f$ ) through the PI controller. This error frequency is further added to the frequency of the fundamental component to get the correct frequency of the input ( $f_{new}$ ) which is used to calculate the phase correction ( $\theta_{FLL}$ ) for the synchronization of the reference signal with the input signal. These parameters are used to generate new reference signals with a frequency the same as the frequency of the input signal. The equation used to calculate the  $\theta_{FLL}$  is given below:

$$\theta_{FLL} = \int_0^{2\pi} 2\pi f_{new} dt \quad (4)$$

The updated frequency of the input signal is used by the

proposed OWFFT with optimal resolution. Equation (5) shows the general equation for the FFT. Where ‘ $k'$ ’ is the spectral point and ‘ $N$ ’ is the size of the FFT.

$$X(k') = \sum_{r=0}^{\frac{N}{2}-1} x(2r) e^{-\frac{j4\pi r k'}{N}} + e^{-\frac{j2\pi k'}{N}} \sum_{r=0}^{\frac{N}{2}-1} x(2r) e^{-\frac{j4\pi r k'}{N}} \quad (5)$$

Sampling frequency and the FFT size are defined before taking the FFT which defines the resolution. Equation (6) shows the resolution of FFT. Where ‘ $f_s$ ’ is the sampling frequency.

$$\Delta f = \frac{f_s}{N} \quad (6)$$

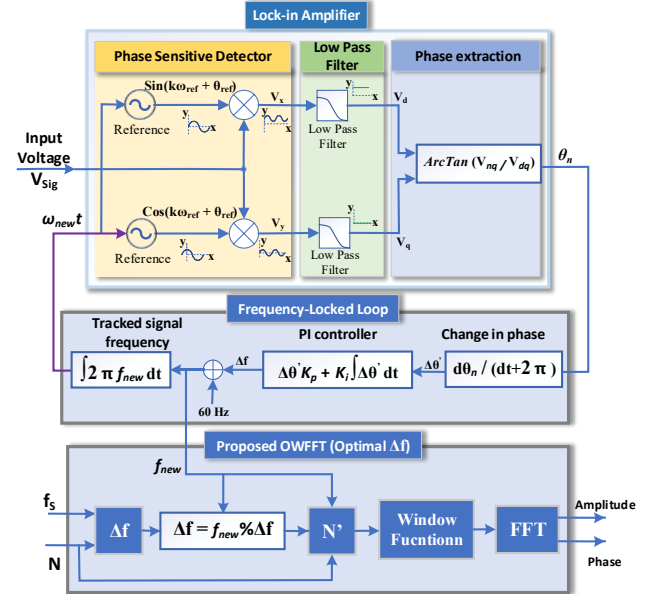


Fig. 3. Block diagram for the proposed OWFFT with LIA-FLL.

The error in the FFT result will be approaching zero if the input signal has a frequency component with a frequency equal to the integer multiple of the resolution. Equation (7) shows this condition. Where ‘ $f_{sig}$ ’ is the frequency of the input signal and  $k' = 1, 2, 3, \dots, n$ .

$$k' \times \Delta f = f_{sig} \quad (7)$$

Now, by using this condition the relationship is derived to get the optimized number of samples to get the signal frequency to be equal to the integer multiple of resolution. Equation (8) shows the derived relationship between input signal frequency and the optimized size of the FFT. Where ‘ $N'$ ’ is the calculated size.

$$N' = \frac{k' f_s}{f_{sig}} \quad (8)$$

The signal frequency is obtained from the LIA-FLL so the ( $f_{sig} = f_{new}$ ). Equation (9) shows the remainder of  $f_{sig}$  that is used to decide the selection of number of samples. Equation (10) shows the remainder obtained by dividing the calculated number of samples. If the remainder is greater than one, then a one is added to the calculated number of samples to get the even number of samples as FFT needs an even number of samples.

$$R1 = f_{new} \% \Delta f \quad (9)$$

$$R2 = N' \% 2 \quad (10)$$

$$R2 = \begin{cases} > 1 : N' = N' + 1 \\ \leq 1 : N' = N' \end{cases} \quad (11)$$

After the calculation of the optimized resolution, a window function is applied further to improve the accuracy of the results. In this paper, the Hanning window is selected for the experiment. Equation (12) shows the expression used for the Hanning window. Where ‘ $n$ ’ is the discrete sample number and ‘ $N$ ’ is the size of the window.

$$w(n) = 0.5 \left[ 1 - \cos\left(2\pi \frac{n}{N}\right) \right] \quad (12)$$

After the application of the window function, the FFT results are obtained in the form of the phase and amplitude for the frequency components present in the input signal.

### 3. Experimental Results and Discussion

The validation of the proposed method OWFFT with LIA-FLL for harmonic detection is carried out by experimental implementation using LabVIEW and NImyDAQ. The harmonic profile with per unit voltage is defined as (Fundamental 1pu, 2<sup>nd</sup>h 0.1pu, 3<sup>rd</sup>h 0.1pu, 5<sup>th</sup>h 0.1pu, 7<sup>th</sup>h 0.1pu). The frequency of the fundamental component is set to be 60Hz. For evaluation under frequency dynamics, the computation is performed over a range of (59Hz - 61Hz). For FFT computation the initial resolution of 1Hz ( $\Delta f = 1\text{Hz}$ ) and the Hanning window function is selected respectively.

Table 1. Comparison of Maximum Absolute Amplitude Estimation Error

k	Maximum Absolute Amplitude Estimation Error (%) with Frequency Deviation (59Hz-61Hz)			
	(Proposed) OWFFT	Hanning WIFFT [3]	Hanning WDIFFT [2]	Nuttall double-window [1]
Fund	1.4952E-5	2.8620E-1	3.00E-2	3.88E-3
2 <sup>nd</sup>	5.8435E-5	-----	3.10E-1	3.63E-4
3 <sup>rd</sup>	1.3394E-4	5.1059E-1	2.40E-1	1.20E-3
5 <sup>th</sup>	3.7349E-4	1.3920E0	3.10E-1	3.10E-1
7 <sup>th</sup>	7.3174E-4	1.7049E0	2.40E-1	2.40E-1

Table 2. Comparison of Maximum Absolute Frequency Estimation Error

Maximum Absolute Frequency Estimation Error (Hz) with Frequency Deviation (59Hz-61Hz)			
(Proposed) OWFFT	Hanning WIFFT [3]	Hanning WDIFFT [2]	Nuttall double-window [1]
0.0002	0.006	0.0004	0.009

The experimental results and comparison with the currently available methods shows that the proposed method is the most optimized method for amplitude and frequency estimation in presence of harmonics and frequency deviations with the maximum absolute amplitude estimation error of  $\pm 0.00073\%$  and maximum absolute frequency estimation error of  $\pm 0.0002\text{Hz}$ . The figure 4 plot also verified the fact that the error is minimum if the input signal frequency is an integer multiple of resolution (60Hz, 59Hz, and 61Hz) and the error is maximum if the input signal frequency is the non-integer multiple of the resolution (59.3Hz and 60.7Hz)

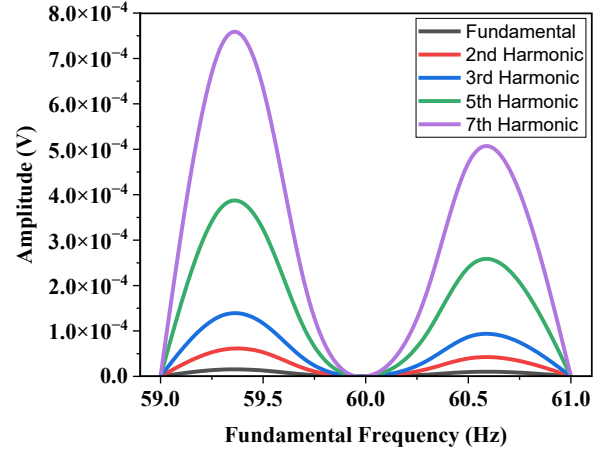


Fig. 4. Maximum Absolute Amplitude Estimation Error (%) for proposed OWFFT with LIA-FLL

### 4. Conclusion

In this paper, a novel method is proposed for power harmonic analysis. This method is based on the optimized windowed FFT (OWFFT) with adaptive resolution and Lock-in Amplifier Frequency-Locked Loop (LIA-FLL). The LIA-FLL tracked the dynamics in the fundamental frequency of the input signal. This tracked frequency is used to calculate the optimized resolution which is used by the windowed FFT to get the signal information accurately. The validity of the proposed method is verified by the experimental implementation of the algorithm using NImyDAQ and LabVIEW environment with an input signal consisting of five frequency components (Fundamental 1pu, 2<sup>nd</sup>h 0.1pu, 3<sup>rd</sup>h 0.1pu, 5<sup>th</sup>h 0.1pu, 7<sup>th</sup>h 0.1pu). The fundamental frequency of the input signal is varied in the range of (59Hz – 61Hz). The absolute error for amplitude extraction and frequency estimation is compared with the currently available method (Hanning window interpolated FFT, Hanning window double-spectrum-line Interpolated FFT, and Nuttall double-window FFT). Experimental results shows proposed method has maximum absolute amplitude estimation error of  $\pm 0.00073\%$  and maximum absolute frequency estimation error of  $\pm 0.0002\text{Hz}$ .

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